On the Security of Keyed Hashing

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Radboud University (The Netherlands)

ISC Winter School on Information Security and Cryptology, February 24, 2021

Outline

Deck functions and some modes

An alternative for block-cipher based crypto

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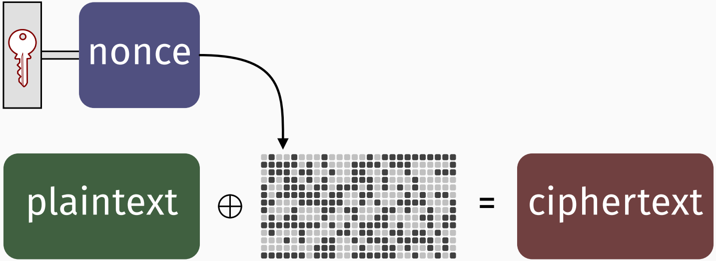
Security of mode with concrete F_K

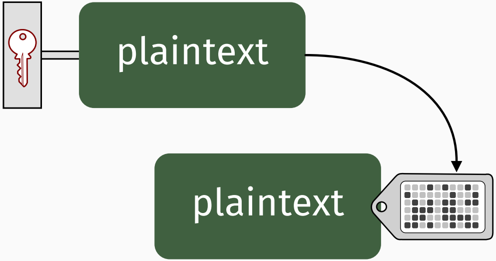
Breaking probability $\leq \epsilon_m(M, N) + \epsilon_p(M, N)$

Stream encryption: short input, long output

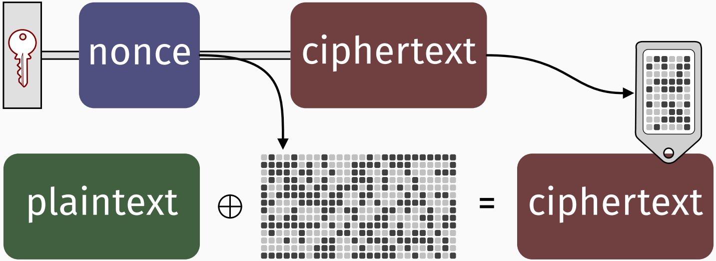


$$C \leftarrow P + F_K(N)$$

MAC computation: long input, short output



$$T \leftarrow 0^t + F_K(P)$$



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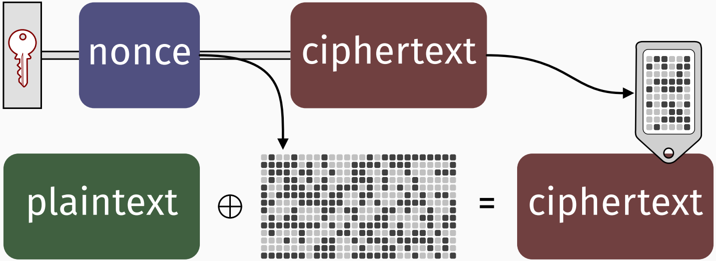
Authenticated encryption (AE)



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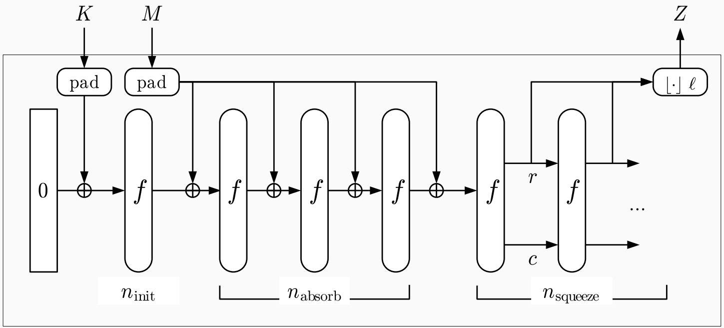
…and much more, see Youtube video of All on Deck! [Keccak Team, RWC 2020]

`https://www.youtube.com/watch?v=CQDsLhf-d-A` at minute 30

How to build a deck function?

A serial deck function construction: donkeySponge [KT, DIAC 2012]



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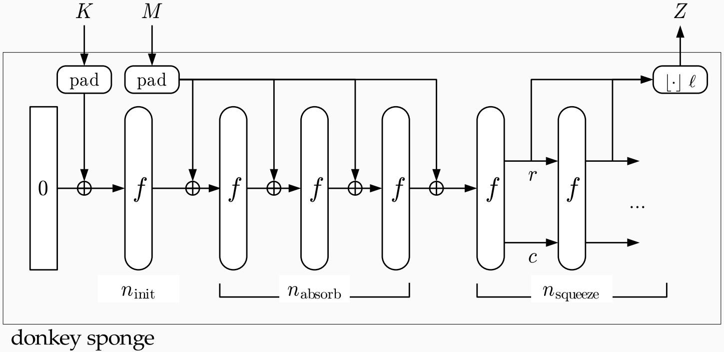
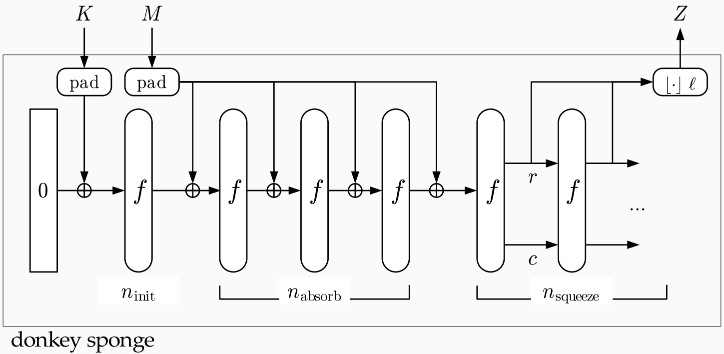


Sponge with secret key K as initial state



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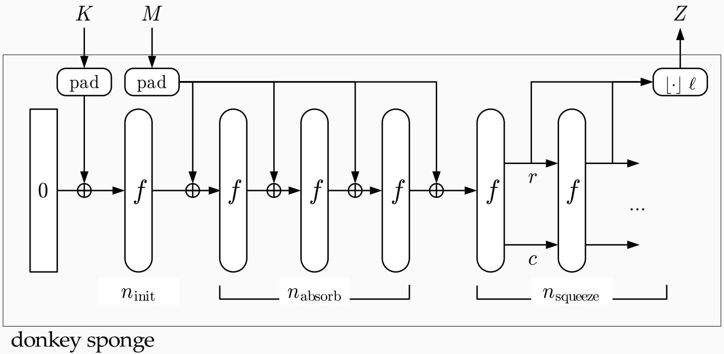

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- Expansion of state to output stream: standard sponge squeezing

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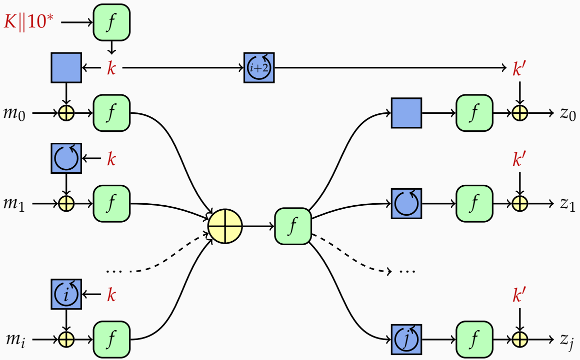

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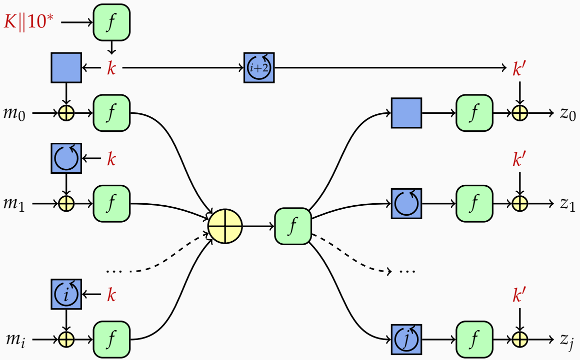

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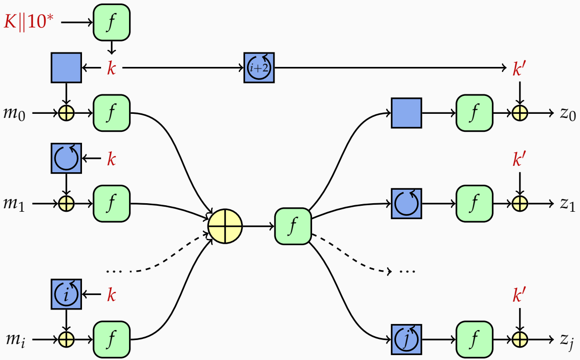

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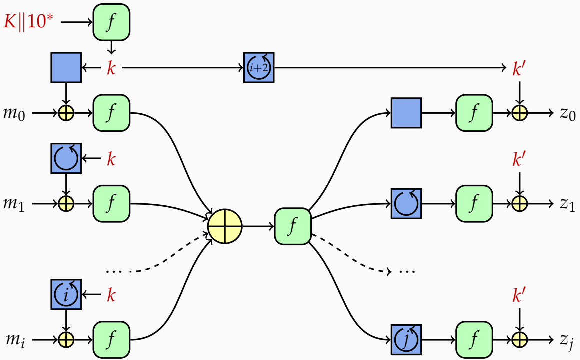

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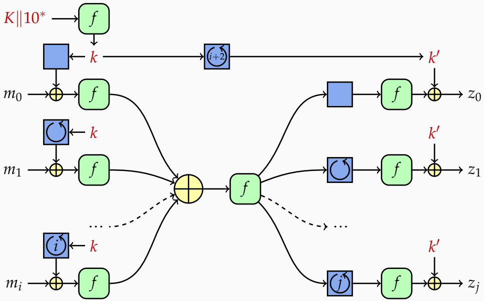
- Expansion: *rolling state* filtered by f and secret mask



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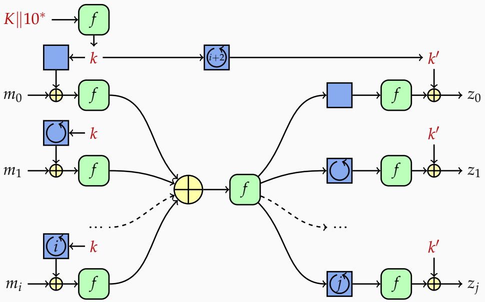

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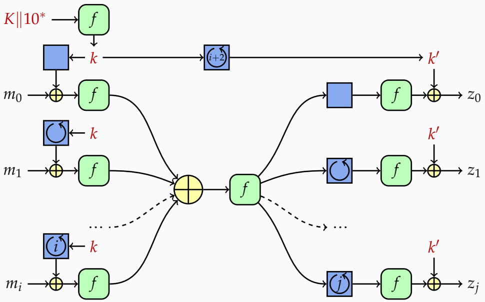

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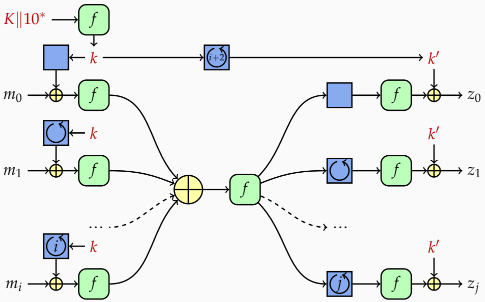

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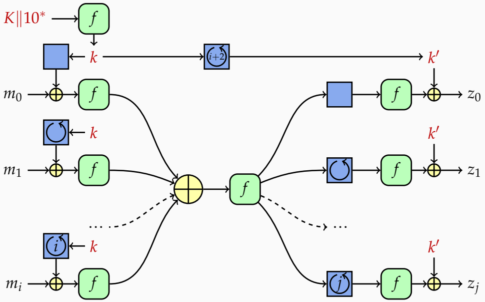

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Keyed hashing

Keyed Hashing

- $F : \mathcal{K} \times \mathcal{M} \to \mathcal{A}$
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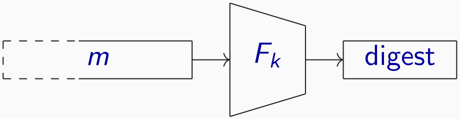
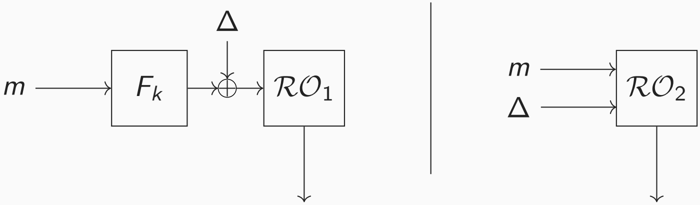
- Convention: F_k denotes F with a fixed key $k \in \mathcal{K}$



Security notion: blinded keyed hash security



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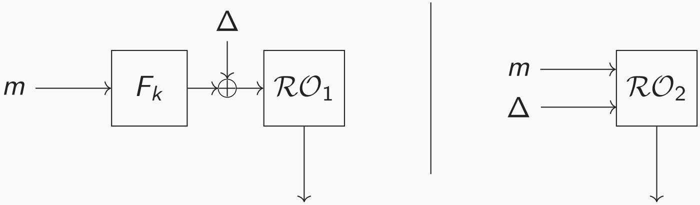

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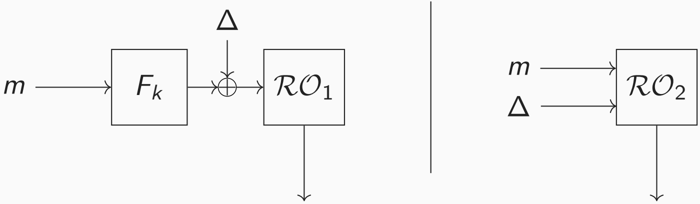

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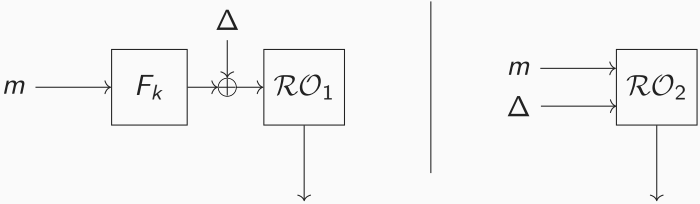

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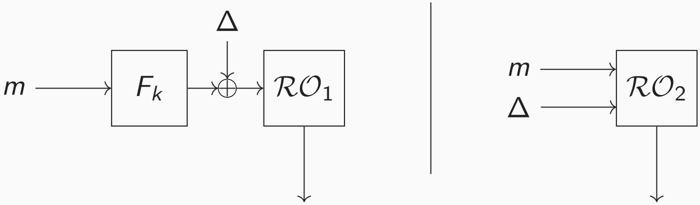

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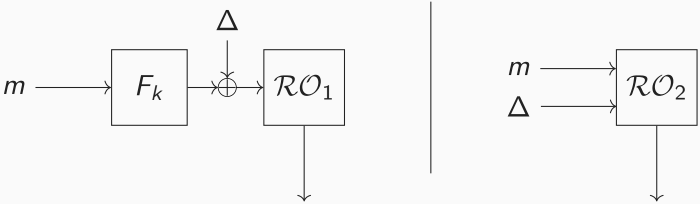
  - adaptability does not help so attacker can fix queries in advance

Blinded keyed hash security and deck functions



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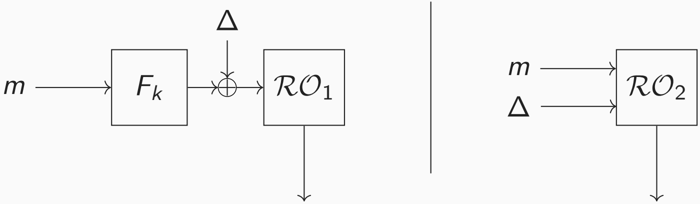

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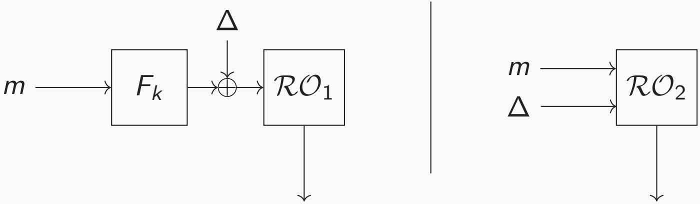

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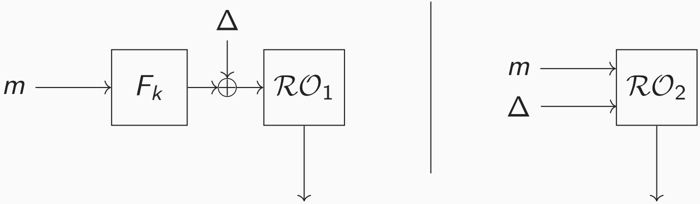

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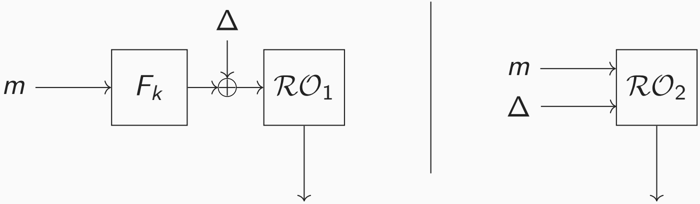

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- We study the collision probability of sets of queries AKA *message sets* D

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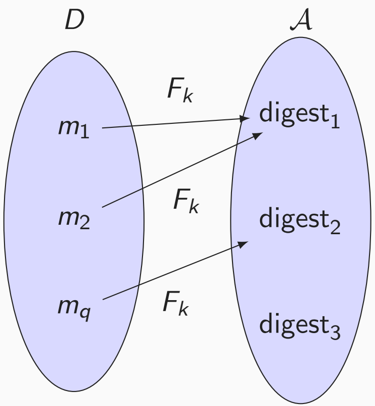

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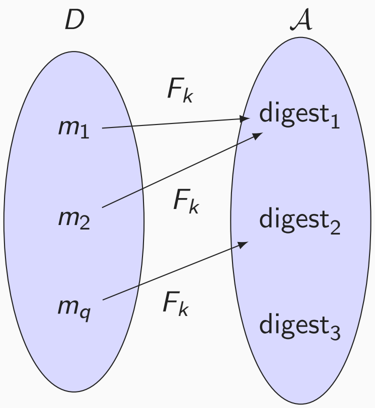

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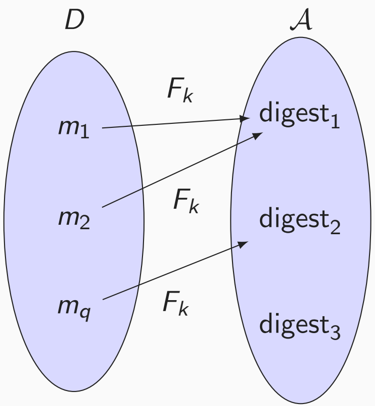

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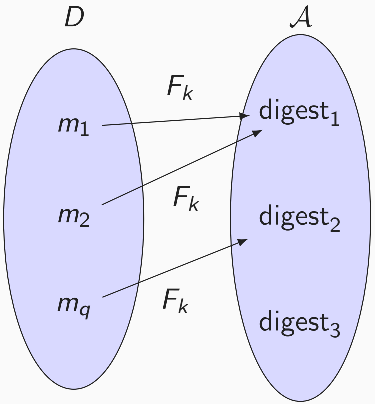

Graphical view

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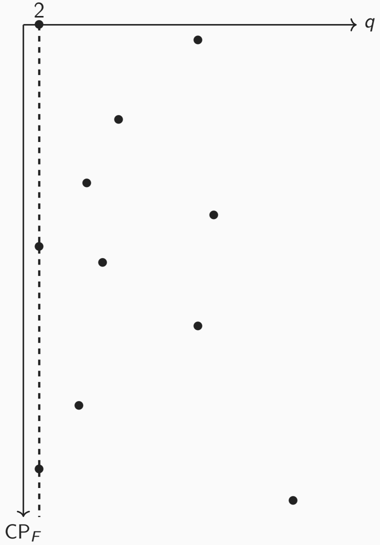

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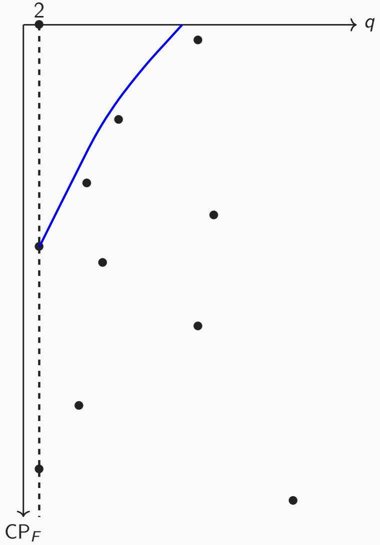

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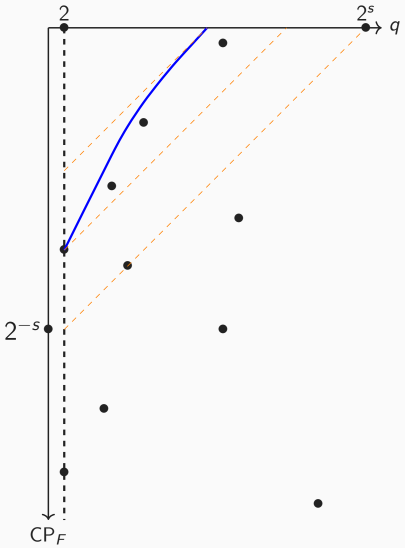

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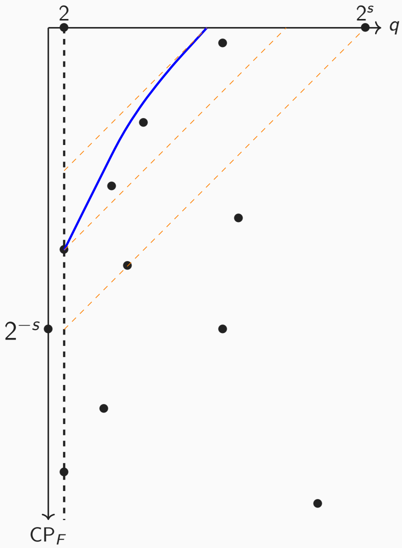

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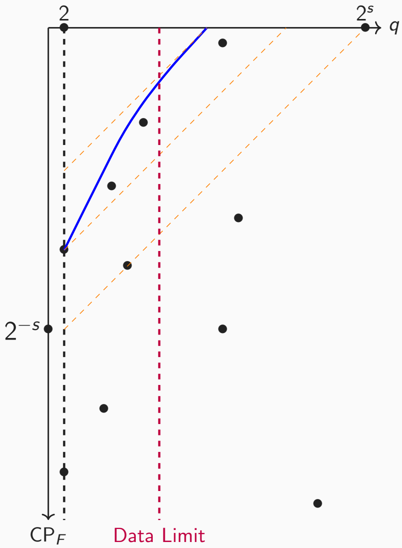
- In real-world settings q may be limited



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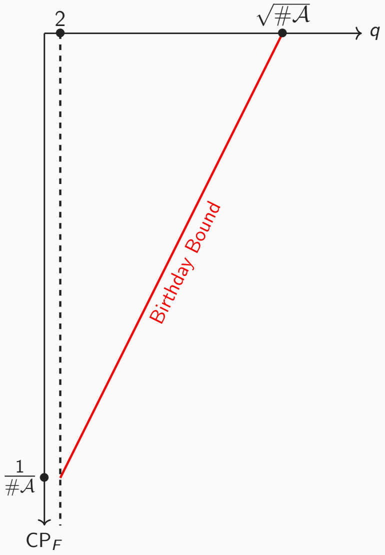

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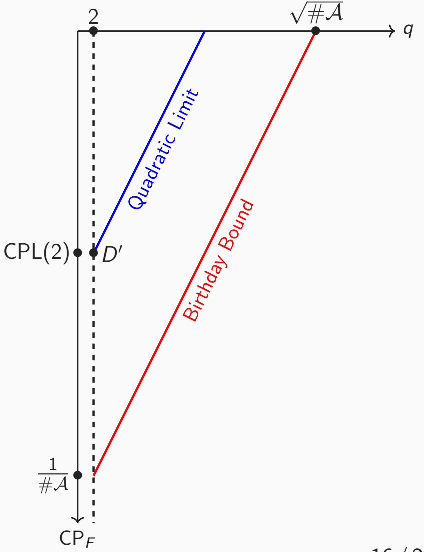

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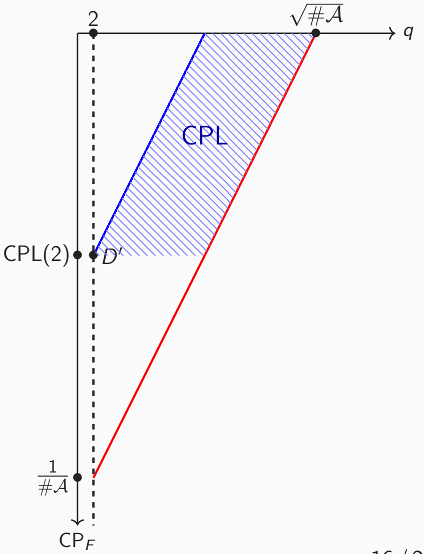

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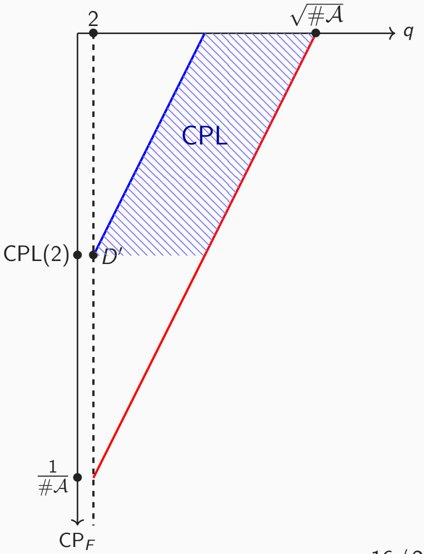
- Equality on two conditions:



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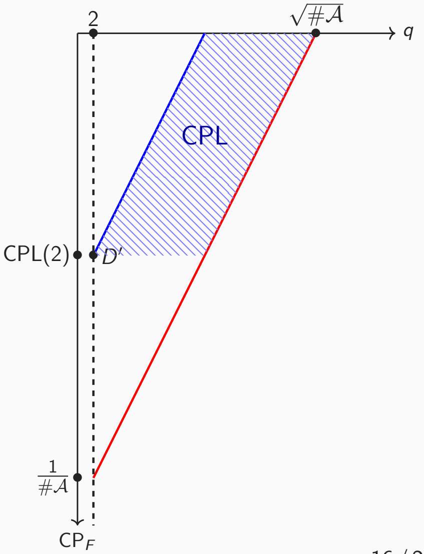

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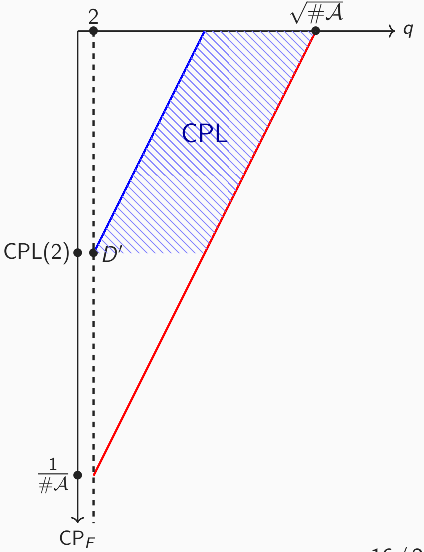

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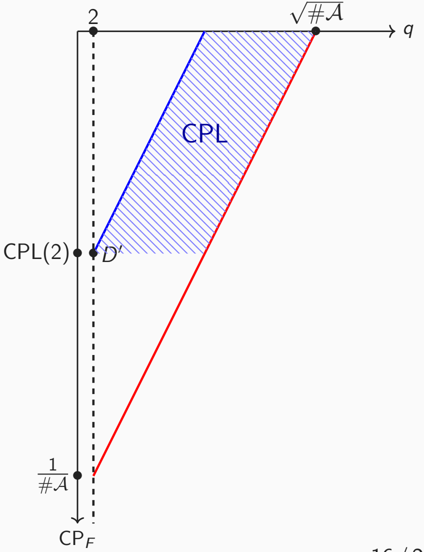

Two concrete constructions

Serial construction

- From a *block function* $f : G \to G$ ...
- we build a keyed compression function $F_{\text{serial}} : \mathcal{K} \times \mathcal{M} \to \mathcal{A}$ with
  - $\mathcal{K} = G^\kappa$
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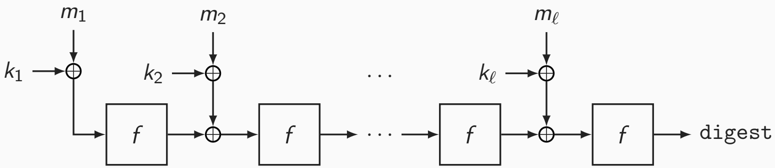

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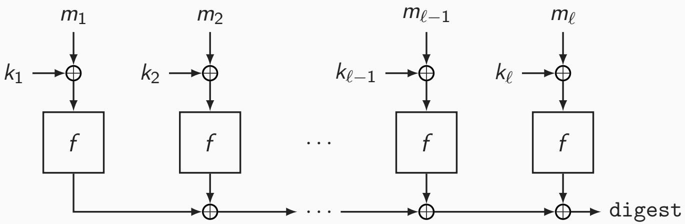

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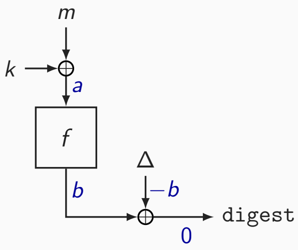

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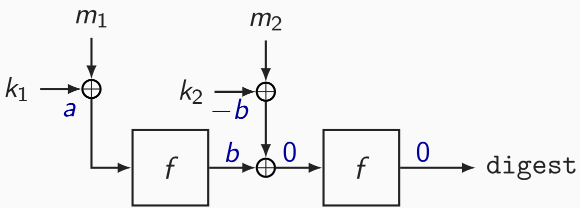

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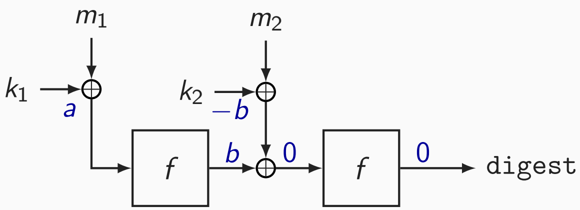

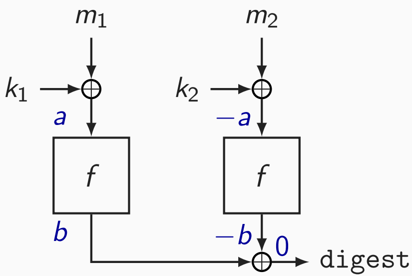

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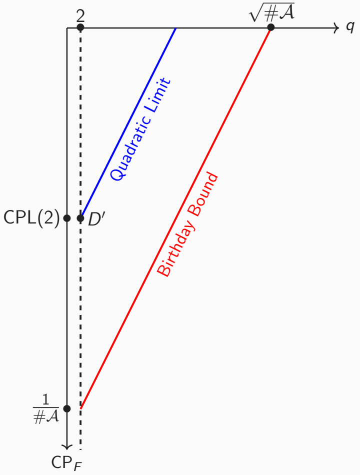

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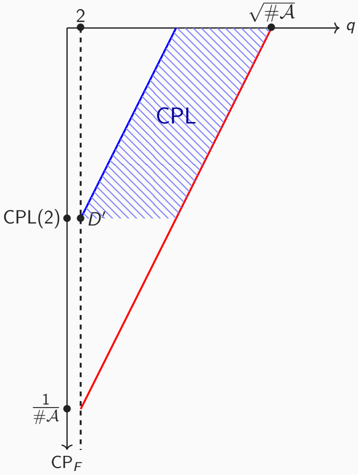

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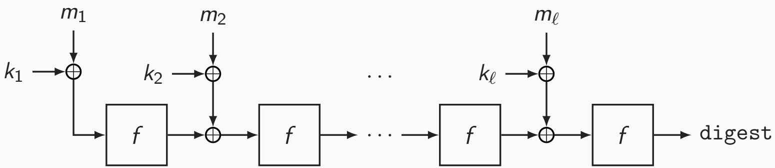

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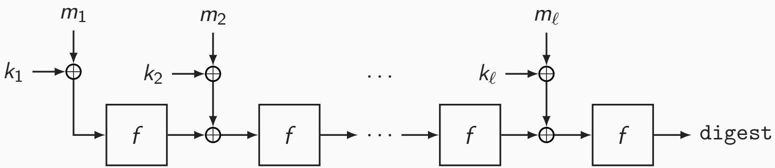

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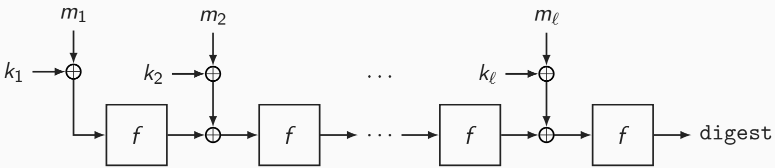

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Lemma (offset-invariance)

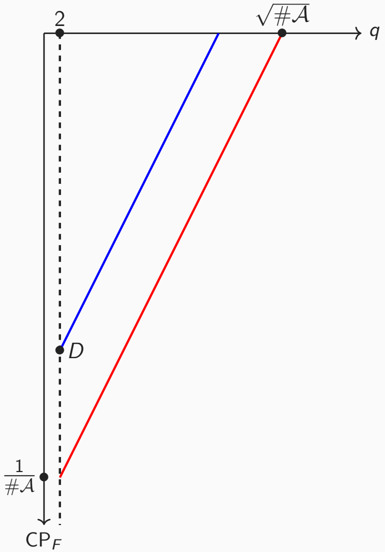
The collision probability is invariant under an offsets of the message set

$$\forall \mu \in G^\kappa : \mathrm{CP}_F(D + \mu) = \mathrm{CP}_F(D)$$

Linear extension of a message set

For $D' = D \cup (D + \mu)$ with μ random

$\mathrm{E}(\mathrm{CP}(D')) \geq 1 - (1 - \mathrm{CP}(D))^2 \approx 2\mathrm{CP}(D)$



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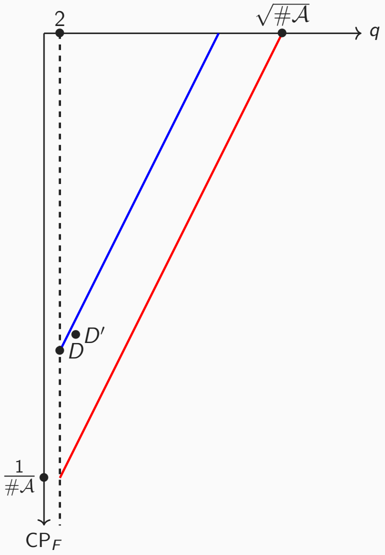

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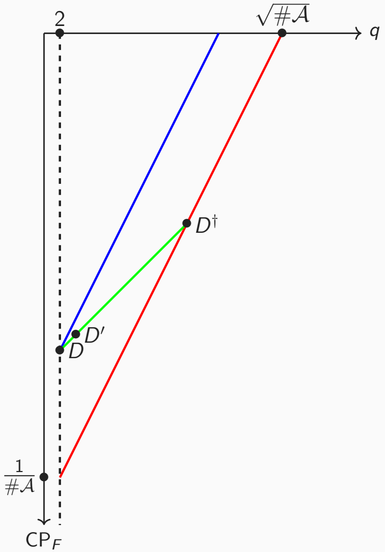

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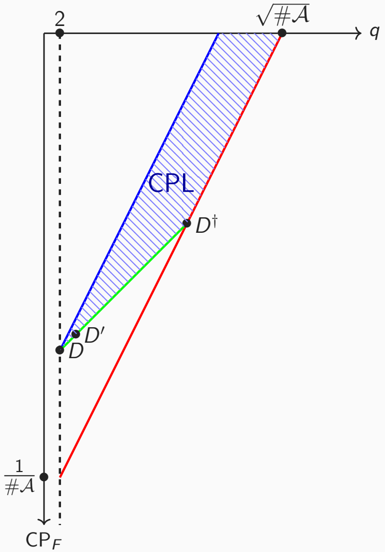

Choosing the block function

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- Did we really come all this way to fall back on block ciphers?

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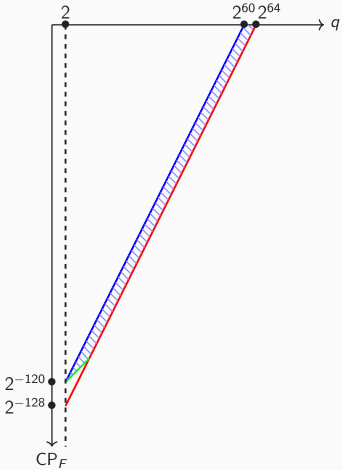

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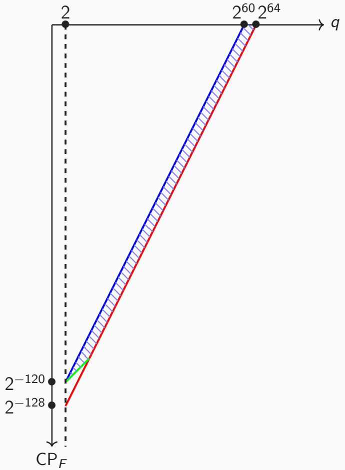

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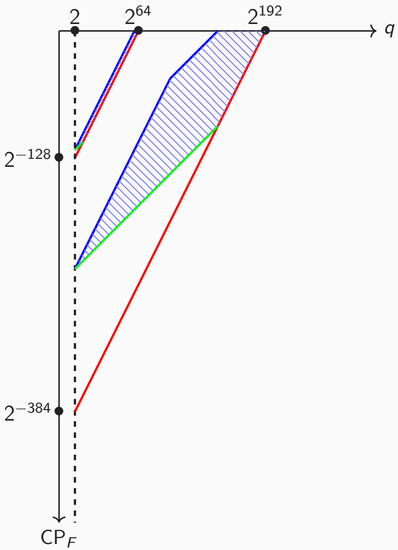

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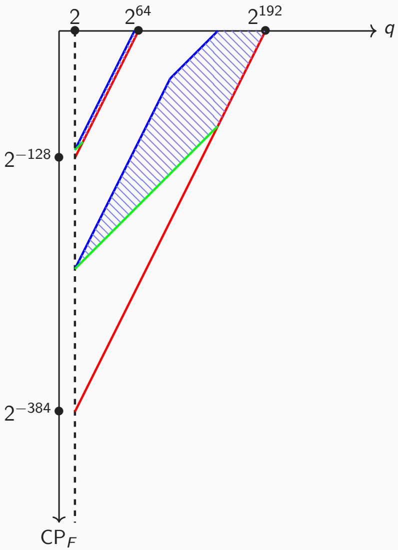

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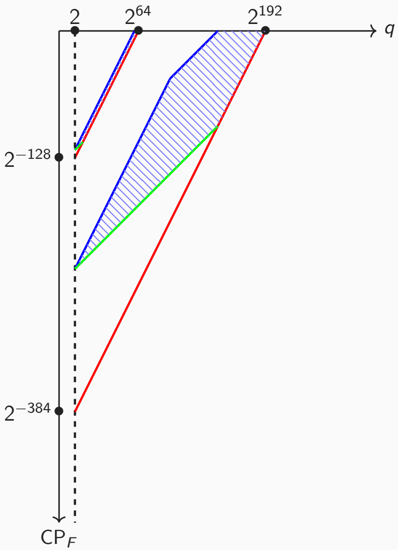
is it fair to compare 6R XOODOO with 4R AES?

- 4R AES takes about 3 times more operations per bit than 6R XOODOO



Conclusion

Keyed hashing with wide permutations

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- parallel construction outperforms serial construction

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Thank you for your attention!